sectional area of metal skeleton per m^2; h, distance between centers of openings; P, wetted perimeter of channels per m^2 of cross section; $Bi_\delta = \alpha P\delta^2/\lambda f$, Biot number; $g = \rho(1 - f)uc_p\delta/\lambda f$, dimensionless fluid flow rate; t_M, dimensional metal temperature; t_F, dimensional fluid temperature; t_{0M}, t_{1M}, t_{0F}, initial temperature of metal and surfaces and inlet temperature of fluid.

LITERATURE CITED

1. V. P. Esachenko, V. A. Osipova, and A. S. Sukomel, Heat Transfer [in Russian], Énergiya, Moscow (1975).
2. G. Korn and T. Korn, Mathematical Handbook for Scientists and Engineers, McGraw-Hill, New York (1968).

REGENERATIVE HEAT EXCHANGERS WITH PERIODIC TIME VARIATION OF COOLANT TEMPERATURE

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Analytic expressions are derived for the temperatures of a heat-exchanger solid and coolant.

Operating elements of power plants and chemical reactors are frequently subjected to the action of a coolant whose temperature varies periodically with time. The change in temperature of the solid parts of these elements determines the service characteristics of the plant or reactor, e.g., the change in the rate of catalytic reactions, etc. From the point of view of heat transfer such installations are regenerative heat exchangers.

Let us consider a heat exchanger whose heat-retaining elements have such characteristics that the temperature gradient in elements of the solid can be assumed negligibly small. In addition, we assume that the gas or liquid (from now on for brevity we refer only to a gas) moves through the free space of the heat exchanger of length L in such a way that the temperature of the gas or solid is the same at all points of a cross section of the heat exchanger perpendicular to the direction of gas flow.

Then, following [1], the problem is reduced to that of solving the following system of equations:

$$\frac{\partial T_1}{\partial t} = a(T_2 - T_1), \tag{1}$$

$$\frac{\partial T_2}{\partial x} + \theta\frac{\partial T_2}{\partial t} = b(T_1 - T_2), \tag{2}$$

where $a = \alpha Al/c_1Ml$; $b = \alpha Al/c_2\dot{m}$; $\theta = \rho_2 Vl/\dot{m}$; $0 \leqslant t > \infty$.

The boundary and initial conditions follow from the formulation of the problem:

$$T_1(x,\ t)|_{t=0} = 0,\ T_2(x,\ t)|_{t=0} = 0, \tag{3}$$

$$T_2(x,\ t)|_{x=0} = D\sin\omega t. \tag{4}$$

In most treatments of regenerative heat exchangers the second term on the left-hand side of Eq. (2) is assumed negligibly small in comparison with the first. To obtain a more general solution we retain this term in the initial system of equations.

After taking Laplace transforms of Eqs. (1)-(4) we have

$$p\bar{T}_1 = a(\bar{T}_2 - \bar{T}_1), \tag{5}$$

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$$\frac{\partial \overline{T}_2}{\partial x} + \theta p \overline{T}_2 = b(\overline{T}_1 - \overline{T}_2), \tag{6}$$

$$\overline{T}_2(x, p)|_{x=0} = D \frac{\omega}{p^2 + \omega^2}. \tag{7}$$

From Eqs. (5)-(7) we obtain

$$\overline{T}_1 = \frac{a}{p+a} \overline{T}_2, \tag{8}$$

$$\overline{T}_2 = D \frac{\omega}{p^2 + \omega^2} \exp\left[-\left(\frac{bp}{p+a} + \theta p\right) x\right], \tag{9}$$

$$\overline{T}_1 = D \frac{\omega}{p^2 + \omega^2} \frac{a}{p+a} \exp\left[-\left(\frac{bp}{p+a} + \theta p\right) x\right], \tag{10}$$

from which, by using Eqs. (23.69), (20.2), (23.21), and (20.8) of [2], we obtain

$$T_1 = Da \int_0^t \sin[\omega(t - \tau - \theta x)] \exp[-(a\tau + bx)] I_0(2\sqrt{abx\tau})\, d\tau, \tag{11}$$

$$T_2 = D\omega \int_0^t \{\cos[\omega(t - \theta x - \tau)] + (a/\omega) \times \sin[\omega(t - \theta x - \tau)]\} \exp[-(a\tau + bx)] I_0(2\sqrt{abx\tau})\, d\tau. \tag{12}$$

Here $I_0(z)$ is the zero-order modified Bessel function of the first kind. It is easy to see that

$$T_1(x, 0) = 0,\ T_2(x, 0) = 0,\ T_2(0, t) = D \sin \omega t.$$

From the expressions obtained for the temperatures of the solid and gas it is clear that for systems in which there are steady temperature oscillations the "lag" determined by the nonstationary term on the left-hand side of Eq. (2) is small. Thus, if we consider flow through a layer of dispersed material in a device of radius R,

$$\theta = \frac{\rho_2 V_l}{m} = \frac{\rho_2 \pi R^2 (1-\varepsilon)}{u_2 \rho_2 \pi R^2} = \frac{1-\varepsilon}{u_2},$$

i.e., for a gas velocity u_2 = 1-10 m/sec, θ = 0.5-0.05 sec/m.

When the solid is a layer of spherical particles of radius r, we have $a = 3\alpha/c_1 r \rho_1$, $b = 3\alpha\varepsilon/rc_2 u_2 \rho_2$. The length of the heat exchanger over which the temperature perturbations specified at the initial cross section diminish by a factor e is of the order 1/b.

When at x = 0

$$T_2(x, t)|_{x=0} = D, \tag{13}$$

we have

$$T_1(x, t) = Da \int_0^t I_0[(2\sqrt{abx(\tau - \theta x)})] \exp[-[a(\tau - \theta x) + bx]]\, d\tau, \tag{14}$$

$$T_2(x, t) = DI_0(2\sqrt{abx(t - \theta x)}) \exp\{-[a(t - \theta x) + bx]\} + T_1(x, t). \tag{15}$$

In the general case, when an arbitrary time variation of the gas temperature is specified at the boundary x = 0

$$T_2(x, t)|_{x=0} = F(t), \tag{16}$$

then

$$T_1 = a \int_0^t F(t - \tau) I_0[2\sqrt{abx(\tau - \theta x)}] \exp[-[a(\tau - \theta x) + bx]]\, d\tau, \tag{17}$$

$$T_2 = T_1 + \frac{1}{a} \frac{\partial T_1}{\partial t}. \tag{18}$$

NOTATION

A_l, solid-gas heat-transfer surface; V_l, volume of heat exchanger free of solid particles; M_l, mass of heat-exchanger solid; subscript l indicates that the quantity refers to a unit length of the heat exchanger; c_1, specific heat of solid; $\dot{m}$, mass flux of gas through heat exchanger; c_2, specific heat of gas; ρ_2, density of gas; T_1, temperature of solid; T_2, temperature of gas at point x; α, solid gas heat-transfer coefficient.

LITERATURE CITED

1. E. R. Eckert and R. M. Drake, Analysis of Heat and Mass Transfer, McGraw-Hill, New York (1959).
2. V. A. Ditkin and A. P. Prudnikov, Handbook of Operational Calculus [in Russian], Vysshaya Shkola, Moscow (1965).

MATHEMATICAL MODEL OF THE ELECTROCONTACT HEATING OF A STEEL BAR IN THE REGION OF CURRENT-CONDUCTING ELECTRODES

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A mathematical model of the electrocontact heating of a steel bar is developed, allowing the temperature fields in any cross section to be calculated. The adequacy of the model is verified by comparison with experiment.

The mathematical model of electrocontact heating (ECH) of steel bars proposed in [1] allows the temperature field to be calculated only in cross sections of the bar from its ends. In the present work, an attempt is made to develop this mathematical model of ECH so as to admit the possibility of calculating the temperature field in any cross section.

In cross sections far from the ends of the bar, axial heat currents may be neglected, and the temperature field is two-dimensional. Close to the ends of the bar, axial heat currents cannot be neglected. Therefore, it is necessary to solve the three-dimensional equation of heat conduction with internal heat sources

$$C_s D_s \frac{\partial t}{\partial \tau} = \lambda_s \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) + \frac{\partial \lambda_s}{\partial x} \frac{\partial t}{\partial x} + \frac{\partial \lambda_s}{\partial y} \frac{\partial t}{\partial y} + \frac{\partial \lambda_s}{\partial z} \frac{\partial t}{\partial z} + q_V. \tag{1}$$

The heat transfer by convection and radiation occurring at the surface of the bar is taken into account as in [1]. Heat transfer between the heated bar and the water-cooled copper current-conducting electrodes is determined by two competing mechanisms: heat supply as a result of contact heat transfer and the liberation of additional heat in the region of the contacts because there is a transient electrical resistance between the contact electrode and the bar. In this case the boundary condition takes the form

$$-\lambda_s \frac{\partial t}{\partial n}\bigg|_{\text{sur}} + Q_C = \alpha_{CH}(t_{BS} - t_{CS}). \tag{2}$$

For contact heat transfer, α_{CH} is found from the formula [2]

$$\alpha_{CH} = 1.6 \cdot 10^4 \frac{\lambda_s \lambda_C}{\lambda_s + \lambda_C} \left(\frac{\bar{p}}{3\sigma_A} K \right)^{0.86} + \frac{\lambda_A}{\delta_E}. \tag{3}$$

Here K is a constant taking values from 1 to 3 depending on the roughness of the contacting surfaces. Since the plasticity of the copper is much higher than for steel, the thickness of the equivalent gas gap is [2]

$$\delta_E = 0.6 h_s. \tag{4}$$

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